

# Gravitational Wave (GW) Radiation Pattern at the Focus of a High-Frequency GW (HFGW) Generator and Aerospace Applications

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**Abstract.** The Gravitational Wave (GW) radiation pattern is derived that results from a rod rotating about a pivot, a dumbbell rotating about its central axis, a pair of stars rotating about their orbital focus, or a stationary circular asymmetrical-array of tangentially jerking elements. The three-dimensional shape of the GW radiation pattern is like a dumbbell cross-section having its long axis perpendicular to the plane of motion or along the central axis of the stationary ring of sequentially jerking elements. The center of the radiation pattern is situated at the pivot, orbital-focus, or center of the stationary array. Knowledge of the GW radiation pattern allows for optimum placement of a detector. In the case of High-Frequency Gravitational Waves (HFGWs), in which the diffraction of the GW radiation is less than the dimensions of the ring of jerking elements, the radiation pattern is situated at the center of the ring and represents a focus or concentration point of the HFGWs. The concentration point extends over a diffraction-limited spot having a radius of  $\lambda_{\text{GW}}/\pi$ , where  $\lambda_{\text{GW}}$  is the wavelength of the HFGW. In the case of a superconductor, prior research has shown that the GW wavelength is foreshortened by a factor of about 300. Thus there could be a much more concentrated diffraction-limited flux of HFGW at the focus. It is shown that the efficiency of a HFGW communications link is approximately proportional to the sixth power of the HFGW frequency. Applications to space technology, involving aerospace communications, and Astronomy are discussed.

## INTRODUCTION

In 1915 Einstein theorized a revolutionary *spacetime* fabric or continuum in his general theory of relativity. He called the undulations or waves propagated in this fabric “*gravitational waves*” (Einstein, 1916). He theorized that they propagate at the speed of light and could be generated, for example, by orbiting stars, spinning rods, dumbbells or asymmetrical rims (essentially composed of many dumbbells). Gravitational waves (GWs) can be sensed by, for example, the change in lengths measured by extremely sensitive interferometers, piezoelectric crystals, superconductors, resonance chambers, etc. They have never been directly sensed, however, and some scientists believed that these waves were unobservable artifacts of Einstein’s theory. The indirect evidence obtained by J. H. Taylor (1994) and R. A. Hulse concerning their observations of a contracting neutron-star pair or pulsar PSR 1913+16, which perfectly matched Einstein’s GW theory, garnered them the 1993 *Nobel Prize* and the skepticism concerning GWs evaporated. According to a set of definitions provided in Chapter 3 of the basic text by Hawking and Israel (1979), High-Frequency Gravitational Waves (HFGWs) have frequencies in excess of 100 kHz and have the most promise for terrestrial generation and practical, scientific and commercial applications. The objectives of this paper are to define the radiation pattern and diffraction-limited spot size at the focus of a HFGW generator as well as to discuss the possibility of Fresnel GW reflection and present applications of HFGW to aerospace communications.

## Quadrupole Approximation to GW Power

The power of gravitational-wave generation increases with the square of the HFGW frequency. From an extension of Einstein's General Theory of Relativity the power of a GW generator is given by his quadrupole equation -- an *approximation* to the *power* of GWs that are *generated by a rapid change in acceleration* (conventionally defined as a "jerk"); the quadrupole is *not* the GW generation process itself (Einstein and Rosen (1937)), it is the lowest-order solution to the GW propagation problem and mass motions that have quadrupole moments are the most effective GW generators. The quadrupole approximation to GW power can be phrased as

$$P(r,\Delta f,\Delta t) = 1.76 \times 10^{-52} (2r\Delta f/\Delta t)^2 W, \quad (1)$$

which is the *jerk formulation of the quadrupole equation* as developed in Baker (2001) and in U.S. Patent 6,417,587. In Eq. (1)  $r$  is the radius of gyration,  $m$ , and  $\Delta f$  is the increase in force,  $N$ , over the time interval,  $\Delta t$ ,  $s$ . For a constant mass  $\delta m$ ,  $\Delta f/\Delta t = \delta m \Delta(\text{acceleration})/\Delta t$  so that the equation states that a third time derivative is imparted to the motion of the mass (usually termed a "jerk") such as a piezoelectric metal electrode or vibrating-membrane element, laser target, permanent magnet, etc. in order to generate GWs. For a continuous train of jerks the frequency,  $\nu$ , is  $\nu = 1/\Delta t$ , and Eq. (1) can be phrased as a function of HFGW frequency as

$$P(r,\Delta f,\nu) = 1.76 \times 10^{-52} (2r\nu\Delta f)^2 W. \quad (2)$$

Let us consider a GW communications link. If collimated in a diffraction-limited beam, then the GW flux (watts per square meter) increases inversely with the square of the GW wavelength (that is, produces a more intense, narrower beam) and if the GW beam is intercepted by a diffraction-limited focusing device (in order to focus an intense spot on a detector), then the flux is increased by another inverse square of the wavelength. The frequency is inversely proportional to the wavelength so we have a square times a square for the efficiency of a GW link. The resulting sixth-power relationship (when the square due to the increased power of the generator with frequency (Eq. (2)) is included) is slightly reduced because the sensitivity of many GW detectors decreases with the square root of the GW frequency. Nevertheless, the value of *High Frequency* is manifest. In this same regard it should be recognized that Low-Frequency Gravitational Waves (LFGWs) generated by most astrophysical sources are expected to be detected by interferometric and resonance devices whose technology is TOTALLY different from the technology of high-frequency detector devices -- as different as AC-motor technology is from microwave technology. Thus LFGW detectors such as LIGO, VIRGO, GEO600, LISA, DECIGO (Japan), CEGO (China), etc., are totally irrelevant and *useless* for HFGW detection.

## Waves of Gravity

Please note that a vibrating-mass-produced oscillation (periodic, time-varying change) in a *classical* "gravitational field" (like tidal changes) is *not* a quadrupole-produced "gravitational wave" in the *spacetime* continuum. As an example, a rapidly pulsating-rotating neutron star generates significant gravitational waves (Misner, *et al.*, 1973, p. 984), but no appreciable oscillations in its gravitational field or tides on a nearby object caused by its rotation. On the other hand, a mass dipole generates *no* gravitational waves (please see, for example, Weber (1964) p. 91), but *does* generate oscillations in its gravitational field (Klemperer and Baker, 1957) or "waves of gravity," which perturb other masses and have tidal influence on them. The magnitude of the tidal influence of waves of gravity can be on the order of meters here on Earth whereas the amplitude of dimension distortions caused by most gravitational waves are on the order of small fractions of a proton diameter. A distinction should also be drawn between "gravitational waves" and "gravity waves." Strictly speaking, the latter refers to water waves in which buoyancy acts as a restoring force and has nothing to do with *spacetime*.

## RADIATION PATTERN

The Gravitational Wave (GW) radiation pattern resulting from a rod rotating about a pivot, a dumbbell, or an asymmetrical rim or ring rotating about its central axis (essentially a collection of dumbbells), or a pair of stars or black holes rotating about their orbital focus, or a stationary asymmetrical or segmented circular array of tangentially jerking or shaking elements in sequence is derived below. The three-dimensional shape of the GW radiation pattern is like a dumbbell (or its cross-section or slice; for two opposed masses), having their long axis perpendicular to the plane of motion or along the central axis of the stationary ring array of jerking, energizable elements (Baker and Li, 2005; Woods and Baker, 2005). It is also of interest that some HFGW detectors strongly resemble such a three-dimensional radiation pattern. Moreover, the center of the radiation pattern is situated at the pivot, orbital focus, or center of the stationary array. Knowledge of the GW

radiation pattern allows for optimum placement of a detector or receiver. In the case of High-Frequency Gravitational Waves (HFGWs) generated in the laboratory, in which the diffraction of the GW radiation is much less than the dimensions of the ring of jerking, energizable elements, the radiation pattern is situated in the middle or at the center of the ring and represents a focus or concentration point of the HFGW. The concentration extends over twice diffraction-limited spot area (since GW goes in both directions) having a radius of  $\lambda_{GW}/\pi$ , where  $\lambda_{GW}$  is the wavelength of the HFGWs. From Saleh and Teish (1991), p. 95, Eq. (3.2-17), the focal length is essentially half of the diameter  $D$  of the stationary ring or  $D/2$ . Thus the F-number of the stationary-ring HFGW generator is  $F_{\#} = 1/2$  and the focus spot radius  $R_{spot}$  is:

$$R_{spot} = (2/\pi) \lambda_{GW} (1/2) = \lambda_{GW}/\pi. \quad (3)$$

Following the analysis found on p. 356 of Landau and Lifshitz (1975) we choose the coordinate origin at the center of a ring of jerking masses, the  $x$  and  $y$  axes in the plane of the ring and the  $z$ -axis perpendicular to  $x$  and  $y$  at the origin. We choose any two diametrically opposed masses, which emulate a pair of orbiting masses,  $m_1$  and  $m_2$ . We have for the radius vectors of the two masses:

$$\mathbf{r}_1 = \{m_2 / (m_1 + m_2)\} \mathbf{r} \quad \text{and} \quad \mathbf{r}_2 = \{m_1 / (m_1 + m_2)\} \mathbf{r}, \quad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2. \quad (4)$$

The components of the mass quadrupole tensor  $D_{\alpha\beta}$ , Eq. (110.11), p. 355 of Landau and Lifshitz (1975), are:

$$D_{xx} = \mu r^2 (3 \cos^2 v - 1), \quad D_{yy} = \mu r^2 (3 \sin^2 v - 1), \quad D_{xy} = 3\mu r^2 \cos v \sin v, \quad \text{and} \quad D_{zz} = -\mu r^2, \quad (5)$$

where  $\mu = m_1 m_2 / (m_1 + m_2)$  and  $v$  is the true anomaly of the emulated orbital motion, that is the polar angle of the radius vector  $\mathbf{r}$  in the  $x$ - $y$  plane between the two diametrically opposed masses. The following analyses are valid according to p. 353 of Landau and Lifshitz (1975): "...at distances ( $r$ ) large compared with the (GW) wavelength of the radiating masses." (In the case of HFGWs,  $r \gg \lambda_{GW}$  ( $10\text{km} > r > 5\text{m}$ ,  $10\text{cm} > \lambda_{GW} > 1\text{pm}$ .) For the emulated circular orbital motion  $r = a$  constant (the semi-major axis,  $a$ ) and the angular time rate of change,  $v'$ , equals the mean motion,  $n$ , so that from Eq. (4.5), p. 87 of Baker (1967),  $n = a^{-3/2} \sqrt{\{k (m_1 + m_2)\}}$ , which we define as  $\omega$ . Following Landau and Lifshitz (1975), we shift to polar coordinates  $\theta$  and  $\phi$ . They consider the two GW polarizations  $e_{\theta\phi} = 1/\sqrt{2}$  and  $e_{\phi\theta} = -e_{\theta\phi} = 1/\sqrt{2}$ . Projecting the tensor  $D_{\alpha\beta}$  on the directions of the spherical unit vectors  $e_{\theta}$  and  $e_{\phi}$ , calculating with Eq. (110.13), p. 355 of Landau and Lifshitz (1975), and averaging over time for a sequence of jerks, we find the result for these two polarization intensities,  $I_1$  and  $I_2$ , and for the sum of intensities  $I = I_1 + I_2$ :

$$\frac{d\overline{I}_1}{d\omega} = (k \mu^2 \omega^6 r^4 / 2\pi c^5) 4 \cos^2 \theta \quad \text{and} \quad \frac{d\overline{I}_2}{d\omega} = (k \mu^2 \omega^6 r^4 / 2\pi c^5) (1 + \cos^2 \theta)^2, \quad (6)$$

so that for the sequential energizing (angular "rate,"  $\omega$ ) of opposed fixed masses around a ring or instantly for any two:

$$\frac{d\overline{I}}{d\omega} = (k \mu^2 \omega^6 r^4 / 2\pi c^5) (1 + 6\cos^2 \theta + \cos^4 \theta). \quad (7)$$

This is the specific GW angular intensity in  $\text{W deg}^{-2}$  (from Landau and Lifshitz, 1975, p. 356) in cross-section that we will now simply denote as  $I(\theta)$  and we will define  $K = (k \mu^2 \omega^6 r^4 / 2\pi c^5)$ . A rather straightforward numerical integration of the specific intensity,  $I(\theta)$ , over a unit-area sphere (one-square-meter-area sphere having a radius of  $1/\sqrt{4\pi} = 0.282\text{ m}$ ) allows for the evaluation of  $K$  for a unit GW source (one watt). Specifically, we approximate each  $10^\circ$  wide zone of colatitude by  $I(\theta)$  evaluated at the center of the zone and multiply this value by the zone area in square degrees ( $10^\circ \times 360^\circ \sin \theta$ ; more convenient here than steradians). We then set the sum of the zone areas multiplied by the appropriate  $I(\theta)$ , set it equal to unity and solve for  $K$ . Thus over the two hemispheres we have (for a sequentially energized circle):

$$2 \sum_{i=5 \text{ to } 85, \text{ step } 10} K (1 + 6 \cos^2 \theta_i + \cos^4 \theta_i) (10^\circ \times 360^\circ \sin \theta_i) = 1 \text{ W} \quad (8)$$

where the colatitude  $\theta_i = i$  in degrees. Solution of Eq. (8) yields  $K = 7.55 \times 10^{-6}$  so that Eq. (7) becomes:

$$I(\theta) = 7.55 \times 10^{-6} P_0 (1 + 6\cos^2 \theta + \cos^4 \theta) \text{ W deg}^{-2} \quad (9)$$

where  $P_0$  is the power of the GW source in watts. One can compute the GW flux at a "cap" on either side of the focus defined by a cone having an altitude,  $D$ , along the  $z$ -axis and a  $\pm 10$  degree ( $20^\circ$ ) vertex angle intersecting the three-dimensional radiation pattern. We choose the  $\pm 10$  degree value rather than the  $\pm 47$  degree half-power points for convenience. By the way, the gain of the cap over the side lobe at  $\theta = 90$  degrees is  $10 \log_{10} (8/1) = 9 \text{ dB}$ . The mean specific intensity over the

cap from Eq. (9) is  $I(5^\circ) = 6.00 \times 10^{-5} \text{ W deg}^{-2}$ . There are  $\pi(10)^\circ = 314.2 \text{ deg}^2$  in the cap so that there is 0.01884 W passing through the cap. The area of the 0.282 m radius unit-area-sphere's cap is about  $\pi \{\sin(10^\circ)(0.282)\}^2 = 0.00743 \text{ m}^2$ . Thus there are  $2.54 \text{ W m}^{-2}$  GW flux through the  $\pm 10^\circ$  cap for each  $P_0 = 1$  watt of power. Therefore the equation for the GW flux, watts per square meter, in either of the caps,  $F_{\pm 10}$ , is given by

$$F_{\pm 10} = P_0 2.54 (0.282/D)^2 \text{ W m}^{-2} \quad (10)$$

where  $D$  = the distance of the detector in either direction from the focus in meters. Clearly, the closer a HFGW detector is to the spot focus the greater the flux. Please note that there is *no* restriction to  $r \ll \lambda_{\text{GW}}$ . Infact, Einstein and Rosen (1937), p. 54, state "At distances ( $r$ ) ... great compared with the wavelengths ( $\lambda_{\text{GW}}$ 's), a progressive wave can be represented with good approximation ..." and on p. 349 of Landau and Lifshitz (1975) they indicate the utility of "... dimensions (of generator) large compared to  $\lambda_{\text{GW}}$  ..." Also Grishchuk (2003) indicated that the relative size of GW generator dimensions and GW wavelength had little or no effect on the accuracy of the quadrupole approximation to GW power.

## **INFLUENCE OF A SUPERCONDUCTOR ON GRAVITATIONAL WAVES**

Consider a HFGW aerospace communications system in which there is a HFGW generator whose emanations are concentrated into a beam by a High-Temperature Superconductor (SC) or HTSC optical system and that at some distance away there is another HTSC optical system to concentrate the HFGW at the detector or receiver. The two optical systems are diffraction limited in their ability to concentrate the HFGW. Such refractive properties, which lead to the concept of HFGW optical systems, were first published by Ning Li and David G. Torr, in 1992 in which on p. 5491 they demonstrated that the phase velocity of a GW,  $v_p$ , was reduced by a factor of about 300 in a superconductor (actually, about a factor, termed the index of refraction,  $N$ , of  $400 \pm 200$ ). This paper was peer reviewed and examined by C. A. Lundquist and Jeeva Anandan, but their results are disputed by Kowitt (1994). Fontana (2004), however, suggests that there is some change in the phase velocity in a SC and an attendant increase in the index of refraction,  $N$ . Thus we will keep the question open. This effect, if it exists, is somewhat similar to that found by Bigelow *et al.* (2003) for light.

### **Application to Aerospace Communications**

As previously noted the area of the conical spread or beam of the HFGW from a generator toward a receiver is inversely proportional to the square of the HFGW frequency or directly proportional to the square of the pulse duration. That is, the higher the HFGW frequency or the shorter the pulse duration the smaller is this area or spread of the HFGW radiation and the greater the concentration (flux) of the HFGW at some distance from the optical system. Another hypothetical optical system, which concentrates the HFGW at the detector or receiver, has a grasp, GW gathering power, or concentration of a maximum value at the focus spot due to diffraction of  $1/\pi$  times the optical-system's objective lens or mirror diameter divided by the GW wavelength all squared. This represents another frequency or pulse-duration squared factor. In sum, when the square of the HFGW frequency,  $\nu$ , of Eq. (2) is included, the hypothetical efficiency of this generator/detector or aerospace transmitter/receiver system is proportional to the sixth power of the GW frequency (see Baker (2001), p. 43) or inversely proportional to the sixth power of the GW pulse duration. In the case of a detector at the focus of the HFGW generator, the HTSC surrounding the focus concentrates the HFGW there (by a factor of the index of refraction squared,  $N^2$ ) since the GW's wavelengths are possibly much shorter in the HTSC and consequently less subject to diffraction. For example, since the HTSC index of refraction for HFGW is speculated to be 300 the HFGW could theoretically be concentrated on to a much smaller diffraction-limited spot area. In fact, the spot area is decreased and the HFGW flux might be increased by a factor of  $(300)^2$ . Due to the speculative very high  $N$ , we will also find (from Eq. (15)) that there can be large Fresnel reflection at the air-HTSC interface.

### **Lenses or Mirrors for Aerospace Communications Applications**

Baker (2003a; 2003b) has discussed fabrication of multifaceted lenses, for aerospace communications applications, composed of a mosaic of several high-temperature superconductors (tiles) such as Yttrium-Barium-Copper-Oxide ( $\text{YB}_2\text{C}_3\text{O}_{7-8}$ ) or YBCO or other media that will refract and focus HFGW. Mirrors for HFGW may also be produced by exploiting the almost total Fresnel reflection at the air-superconductor interface. As an alternative to YBCO, one can utilize a far less expensive HTSC (though somewhat lower temperature, that is lower than the temperature of liquid Helium that allows YBCO's transition to superconductivity) such as steel-clad  $\text{MgB}_2$ . On the other hand, the convenience of YBCO, its easily achieved liquid-nitrogen temperature and its ready availability make it the HTSC of choice. Note that since GW can

pass through all normal matter without attenuation, it can pass through reinforcing lens or mirror structure and even detector apparatus near the focal plane of the lens. Also it has been noted that the wavelength of the gravitational waves in the HTSC is about  $1/N^{\text{th}}$  as long as they are outside the HTSC. There are, however, concerns about possible GW reflection at the air/superconductor interface that should be the subject of future detailed study. But assuming that Fresnel reflection applies, quarter-wavelength anti-reflection having an index of refraction of  $N_r = \sqrt{N_1 N_2}$  (Eq. (7.28), p.170 of Smith (1966)) where the subscripts indicate the medium: sub 1 for the HTSC and sub 2 for air) or about  $N_r = 17$ , must be fabricated and applied to a lens surface. Also, if a superconductor wall or slab is made an exact integer multiple of half a wavelength thick, then the HFGW reflections would cancel. For HFGW with free space wavelength 61mm generated by Film Bulk Acoustic Resonators (Woods and Baker, 2005), the wavelength inside the HTSC will be 61mm/300, so the slab or wall thickness (for example, a HTSC slab surrounding a focal spot to concentrate the HFGW) should be an integer multiple of

$$(\lambda_{\text{GW}}/N)/2 = (61\text{mm}/300)/2 = 102\mu\text{m}. \quad (11)$$

For the X-ray laser generated HFGW (Baker and Li, 2005)  $\lambda_{\text{GW}} \sim 0.029\mu\text{m}$  and the hypothetical HTSC slab thickness around the focal spot would be an integer multiple of  $(0.029\mu\text{m}/300)/2 = 5 \times 10^{-11}$  m. For example, 100,000 integer thicknesses or one  $\mu\text{m}$ . But there are other issues to be raised here, such as whether the HTSC wall or slab can be made at this thickness and with the required uniformity and tolerance. Possibly nano-fabrication or assembly techniques could be utilized to build discrete molecular layers of the  $\text{MgB}_2$  HTSC surrounding the submicroscopic focal spot location.

### **Grasp or Gathering Power of an Astronomical Telescope Lens**

Let us consider an example of a “fast”  $f/0.5$  and 100-meter diameter HTSC mosaic lens for use in an astronomical telescope. Let us also utilize or image an extended celestial source, such as ripples, background “plotches,” or “pixelization” (Hogan, 2002; Brustein, *et al.*, 1995; Gorkavyi, 2003) or other anisotropic features of limited angular extent in the relic or primordial cosmic background. The intensity or power at the focus,  $P$ , for an objective lens diameter,  $d$ , and a focal length,  $f$ , (focal ratio,  $F_{\#} = f/D$ ) and GWflux in  $\text{W m}^{-2}$  is

$$P_0 \approx (\text{GWflux}) (\text{Objective Lens Area}) (1/\{\text{focal ratio}\}^2) \text{ W} \quad (12a)$$

and for  $F_{\#} = 0.5$  (focal length approaching  $D/2$ )

$$P_0 \approx \text{GWflux} (\text{W m}^{-2}) \pi (d/2)^2 (\text{m}^2) (1/0.5)^2 \text{ W}. \quad (12b)$$

Thus the grasp or gathering power, concentration or “gain” for such an extended source with  $d = 100$  m and  $f = 50$  m is  $3 \times 10^4$ . For point sources, such as relatively nearby evaporating primordial mini-black holes (Miller, 2002; Bisnovatyi-Kogan and Rudenko, 2004), the concentration would be much greater. Let us consider the lens system for the 100-meter, extended source configuration. The nominal HTSC index of refraction,  $N$ , utilized in this paper is 300. The lens maker’s equation (please see, for example, Eq. (2.30a), page 35 of Smith (1966)) for a simple spherical-surface thin lens having a front-surface radius,  $R_1$ , and a rear-surface radius,  $R_2$ , is

$$1/f = (N - 1) (1/R_1 - 1/R_2). \quad (13)$$

For a plano-convex lens,  $R_2 \rightarrow \infty$  and with  $N \gg 1$  from Eq. (13)

$$R_1 = Nf = (300)f \quad (14)$$

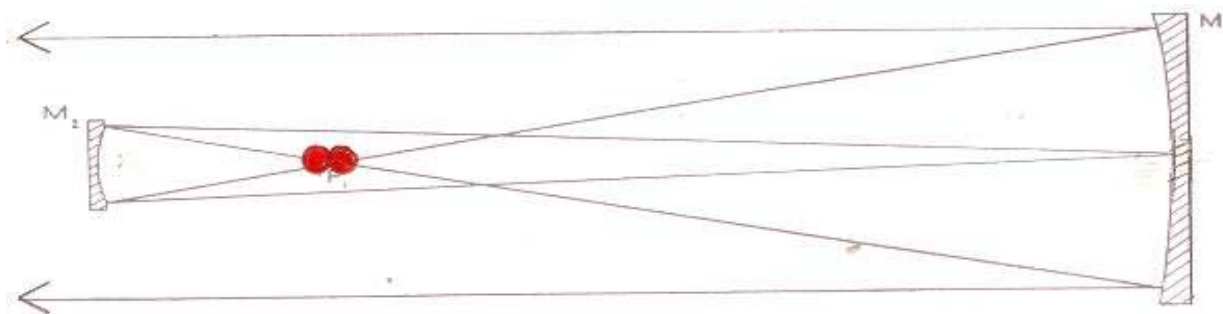
Actually, according to Ning Li (2002) for a superconductor (SC)  $N = c/v_p = 3 \times 10^8 / (1 \pm 0.5) \times 10^6$  or  $N = 300 + 300 - 100$  or  $N = 400 \pm 200$  so that  $R_1$  is about 20,000 m for a nominal lens surface radius. There may exist HFGW dispersion, which is a variation of  $N$  with frequency. Also there exists spherical aberration for a simple spherical lens since the ray angles are so large on the focus side of the lens. Thus, like light-telescope lenses, there may be a need to use a far more sophisticated lens shape or, possibly, multiple lenses, rather than just sections of a sphere, as well as a need for anti-reflecting coatings. In this latter regard, one might utilize a very thin Fresnel-like segmented lens, a multiple of half wavelengths thick, to reduce or avoid Fresnel-lens-surface reflection.

## Reflection or Mirrors for Aerospace Applications

Assuming that the Li-Torr (1992) result is correct, applying conventional optical theory (e.g., Eq. (7.1), p. 145 of Smith (1966)), then there would be strong Fresnel reflection at the air-HTSC interface. Specifically, the fraction of reflected power,  $R$ , at a lens or mirror surface is given by

$$R = (N_1 - N_2)^2 / (N_1 + N_2)^2 = (299)^2 / (301)^2 = 0.987. \quad (15)$$

It appears that Fresnel reflection of GW has not been considered before, but in a superconductor the phase velocity must presumably reduce about  $N = 300$  times; otherwise, we have to postulate that the frequency also changes which seems unlikely. The Fresnel equations (Eq. (15)) are derived purely from phase velocity considerations and group velocity does not enter the derivation, so Fresnel reflection cannot be ignored in our analyses. The utilization of the reflective properties of HTSCs for HFGW-optical systems may be promising. If there is reflection from a superconductor, then one could utilize a “mirror” optical system. Considering the cross-section of the dumbbell-shaped “antenna pattern” of an asymmetrical array of HFGW-generator resonators, X-ray targets, etc., energizable elements, probably a Gregorian-like optical system of two coaxial mirrors as shown schematically in Fig. 1 would be best: a concave paraboloid mirror ( $M_1$ ) on one side of the focus and a much smaller secondary flat or concave ellipsoidal mirror ( $M_2$ ) on the other side, each facing the focus – where the radiation pattern (red; from Eq.(9)) is located. One would need to insure that the mirror geometry was such that the ray path *difference* from either mirror to the focal spot was an integer number of wavelengths so that the HFGWs add in phase. The aerospace communications link would then consist of two sets of such HTSC mirror optical systems facing each other.



**FIGURE 1.** Gregorian-like HFGW Optical Projector (Telescope) System  
(Not to Scale: the Ray Angles Would No Doubt be Greater than the  $\pm 47^\circ$  Half-Power Points)

## Grinding and Polishing the Lenses or Mirrors

The 100 m diameter, astronomical-telescope superconductor’s spherical lens surface is  $20,000 - \sqrt{(20,000)^2 - (100/2)^2} = 6.25$  mm thicker at the center than at the edge of the objective lens and a paraboloid, ellipsoid or flat HTSC mirror for a HFGW reflecting telescope has a surface defined by the details of the mirror-system design. In both cases a mosaic of HFGW tiles rather than a homogeneous, continuous HTSC would be fabricated. A high-temperature superconductor tile, such as YBCO, is brittle and must be treated gently, especially during grinding operations. A fine soft abrasive (softer than that used for glass lenses) is required and soft brass grinding tools should be used in place of the standard cast-iron. Note that a spherical lens surface is readily generated by random grinding and polishing, because any line through the center of the lens sphere is an axis. An ordinary spherical optical surface for light wavelengths is a true sphere to a few millionths of an inch, whereas for, say, one-centimeter GW wavelengths (30 GHz HFGWs),  $1/10^{\text{th}}$  of a mm or 100  $\mu\text{m}$  should be more than ample (more stringent requirements for higher frequencies, of course). These same specifications apply to the HTSC mirror surface. One *ordinarily* would avoid a thickness-to-diameter ratio  $< 1/50^{\text{th}}$  (less than 2 m thickness for a 100-meter diameter lens or mirror) so that the lens or mirror will not spring and warp up during the grinding process. However a strong steel structural backing plate can be utilized to support the multiple-tile, mosaic lens or mirror both during grinding and manufacturing operations as well as when it is deployed operationally (all normal matter, such as steel, is transparent to GW) so that a *much thinner* lens or mirror can be utilized. As discussed on page 417 of Smith (1966) a variation in focal length,  $\Delta f$ , due to a lens-surface radius variation,  $\Delta R_1$ , (say, one tenth of a mm or 0.0001 m), is given by

$$\Delta f = f^2 (N - 1) \Delta R_1 / R_1^2 = (50)^2 (399) (0.0001) / (20000)^2 = 2.5 \times 10^{-7} \text{ m} = 0.25 \text{ } \mu\text{m}, \quad (16)$$

which is very satisfactory. A quarter-wavelength, anti-reflection lens surface coating having a  $\sqrt{N}$  index of refraction (possibly a challenge to fabricate) would need to be applied to the lens surfaces (not an issue for the reflectors).

## SUMMARY AND CONCLUSIONS

Equations have been derived for the radiation pattern and diffraction-limited spot size at the focus of a HFGW generator. The documented influence of a superconductor on dramatically reducing GW phase velocity has been utilized in the design of lenses and mirrors for use in a HFGW aerospace communications system and in a HFGW astronomical telescope. The specifications and fabrication of such telescopes are discussed and a grasp or light gathering power of  $3 \times 10^4$  for anisotropic cosmic sources is calculated. The possibility of Fresnel reflection by a HTSC is analyzed. As discussed in the paper, a HFGW communications signal can pass through all normal matter unattenuated and the power at a receiver is found to be proportional to the sixth power of the GW frequency. Thus a communications system utilizing HFGW would be valuable for communicating over vast interplanetary and interstellar distances and could result in a paradigm change in aerospace communications.

## NOMENCLATURE

a	= semi-major axis of the emulated two-body orbit (m)	$N_r$	= index of refraction for anti-reflecting coating
c	= speed of light ( $\text{m s}^{-1}$ )	$\theta$	= solid angle ( $\text{deg}^2$ )
D	= diameter of the circle of energizable elements or of a lens (m)	$P_0$	= power of a GW generator (W)
$D$	= distance of the detector or receiver from the focus spot of the GW generator (m)	r	= the radius of gyration of an array of asymmetrical jerking, energizable masses or the radius of a circular orbit (m)
$D_{\alpha\beta}$	= mass quadrupole tensor $\alpha, \beta$	$\mathbf{r}$	= $\mathbf{r}_1 - \mathbf{r}_2$ (m)
$D_{xx}$	= tensor component x, x	$\mathbf{r}_1$	= radius vector from origin to $m_1$ (m)
$D_{yy}$	= tensor component y, y	$\mathbf{r}_2$	= radius vector from origin to $m_2$ (m)
$D_{xy}$	= tensor component x, y	$R_1$	= lens front-surface radius (m)
$D_{zz}$	= tensor component z, z	$R_2$	= lens rear-surface radius (m)
$\mathbf{e}_\theta$	= spherical unit vector in the increasing $\theta$ direction	$R_{\text{spot}}$	= radius of the focal spot (m)
$\mathbf{e}_\varphi$	= spherical unit vector in the increasing $\varphi$ direction	R	= fraction of Fresnel-effect GW reflected
f	= focal length of the GW generator or HFGW lens (m)	v	= true anomaly of an emulated two-body orbit (degrees or radians)
$F_{\pm 10}$	= the GW flux at a distance $D$ , from the origin or vertex, of the $\pm 10^\circ$ cap ( $\text{W m}^{-2}$ )	x-axis	= coordinate orthogonal to the y-axis and z-axis in the plane of the energizable elements
$F_\#$	= focal ratio or F-number of a GW generator or HFGW lens	y-axis	= coordinate orthogonal to the x-axis and z-axis in the plane of the energizable elements
$m_1$	= emulated orbital mass 1 or mass of a jerking, energizable element (kg)	z-axis	= coordinate orthogonal to the y-axis and x-axis, perpendicular to the plane of the energizable elements and having origin at its center
$m_2$	= emulated orbital mass 2 or mass of a jerking, energizable element (kg)	$\delta m$	= incremental mass (kg)
I	= intensity of GW (W)	$\Delta f$	= impulsive force on an energizable element (N)
$I$	= specific intensity of GW ( $\text{W deg}^{-2}$ )	$\Delta t$	= incremental time over which $\Delta f$ occurs (s)
k	= gravitational constant for an emulated two-body orbit	$\theta$	= polar, colatitude coordinate (degrees or radians)
K	= normalization constant	$\lambda_{\text{GW}}$	= GW wavelength (m)
n	= mean motion of an emulated two-body orbit	$\mu$	= $m_1 m_2 / (m_1 + m_2)$
N	= index of refraction	v	= frequency of GW ( $\text{s}^{-1}$ )
$N_1$	= index of refraction for GWs in a SC		
$N_2$	= index of refraction for GWs in air		

$v_p$  = speed of gravitational waves in a superconductor,  $\text{ms}^{-1}$        $\omega$  = angular rate of the emulated two-body orbital motion ( $\text{rad s}^{-1}$  or  $\text{deg s}^{-1}$ )  
 $\phi$  = polar, longitude coordinate (degrees or radians)

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